PROBLEMS (uniform angular acceleration)

① A body is rotating with angular velocity of 30 rpm to 75 rpm in 15 seconds. Find the angular acceleration of the body and no. of revolutions made by the body in this 15 seconds.

Solution

$\omega_0 = 30\ rpm = \frac{2\pi \times 30}{60} = \pi\ rad/sec$

$\omega = 75\ rpm = \frac{2\pi \times 75}{60} = 2.5\pi\ rad/sec.$

$t = 15\ sec$

$\omega = \omega_0 + \alpha t$

$2.5\pi = \pi + \alpha \times 15$

$\therefore \alpha = \underline{\underline{0.314\ rad/s^2}}$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= 15\pi + \frac{1}{2} \times 0.314 \times 15^2$

$= \underline{\underline{82.44\ rad}} = \frac{82.44}{2\pi} = \underline{\underline{13.12\ revolutions}}.$

② A wheel is rotating about x axis with a constant angular acceleration of 1.5 rad/s². If the initial & final angular velocities are 7.5 rad/s & 9 rad/s. determine the total angle turned during the time interval in which the change of angular velocity takes place.

Solution

$\omega_0 = 7.5\ rad/sec$; $\omega = 9\ rad/s$, $\alpha = 1.5\ rad/s^2$

$\omega^2 = \omega_0^2 + 2\alpha\theta.$

$9^2 = 7.5^2 + 2 \times 1.5\ \theta.$

$\therefore \theta = \underline{\underline{8.25\ rad}}$

(3) A wheel starting from rest is given an acceleration of 1 rad/s². what will be its speed in rpm at the end of 1.5 minutes. If then, it is uniformily retarded at the rate of 0.5 rad/s², in how many minutes the wheel come to rest.

Solution.

Ist interval.

$\omega_0 = 0$

$\alpha = 1 \text{ rad/s}^2$

$t = 1.5 \text{ min} = 90 \text{ sec.}$

$\omega = \omega_0 + \alpha t$

$= 0 + 1 \times 90$

$= 90 \text{ rad/sec}$

$= \frac{90 \times 60}{2\pi} = \underline{\underline{859.44}} \text{ rpm}$.

IInd interval.

$\omega_0 = 90 \text{ rad/s}$

$\omega = 0$

$\alpha = -0.5 \text{ rad/s}^2$

$\omega = \omega_0 + \alpha t$

$0 = 90 - 0.5t$

$t = 180 \text{ sec}$

$= \underline{\underline{3 \text{ min}}}$

$\therefore$ Total time taken $= 1.5 + 3 = \underline{\underline{4.5 \text{ min}}}$.

(4) A wheel is making 150 rpm and after 10 sec, it is running at 90 rpm. How many revolutions will it make and what time will elapse before it stops, if the retardation is uniform.

Solution

$\omega_0 = 150 \text{ rpm} = \frac{2\pi \times 150}{60} = \underline{\underline{5\pi \text{ rad/s}}}$.

$\omega = 90 \text{ rpm} = \frac{2\pi \times 90}{60} = \underline{\underline{3\pi \text{ rad/s}}}$

$t = 10 \text{ sec.}$

$\omega = \omega_0 + \alpha t$

$3\pi = 5\pi + \alpha \times 10$

$\therefore \alpha = -0.2\pi \text{ rad/s}^2$

$= \underline{\underline{-0.628 \text{ rad/s}^2}}$.

Let T- total time taken when it comes to rest.

$\therefore \omega = \omega_0 + \alpha T$

$0 = 5\pi + (-0.2\pi \times T)$

$\therefore T = 25$ sec

No. of revolutions before it stops:

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$0 = (5\pi)^2 + 2\times(-0.2\pi)\,\theta$

$\theta = 62.5\pi$ rad

No. of revolutions $= \dfrac{62.5\pi}{2\pi} = 31.25$ rev.

⑤ A wheel rotates freely on frictionless bearings at 240 rpm. How many revolutions will it make in 10 sec. Determine the angular speed, if this wheel turns 500 revolutions in 15 sec?.

Solution

$\omega = 240$ rpm $= \dfrac{240\times 2\pi}{60} = 8\pi$ rad/sec

$t = 10$ sec

$\theta = \omega t = 8\pi\times 10 = 80\pi$ radians. (rotates with uniform velocity)

No. of revolutions $= \dfrac{80\pi}{2\pi} = 40$ revolutions

Angular speed when wheel turns 500 revolutions in 15 sec:

$t = 15$ sec

$\theta = 500\times 2\pi = 1000\pi$ rad

$\omega = \theta/t = \dfrac{1000\pi}{15} = 66.67\pi$ rad/sec

$= 209.44$ rad/sec

⑥ A wheel rotating about a fixed axis at 20 rpm is uniformly accelerated for 70 s during which it makes 50 revolutions. Find the angular velocity at the end of this interval and the time required for the speed to reach 100 rev/min.

HW

Solution

$\omega_0 = 20\ rpm = \frac{20 \times 2\pi}{60} = \frac{2}{3}\pi\ rad/sec$

$t = 70\ s;\ \theta = 50 \times 2\pi = 100\pi\ rad$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$100\pi = \frac{2}{3}\pi \times 70 + \frac{1}{2}\alpha (70)^2$

$\therefore \alpha = 0.068\ rad/s^2$

$\omega = \omega_0 + \alpha t$

$= \frac{2}{3}\pi + 0.068 \times 70$

$= 6.85\ rad/s$

Let T be the time taken;

$\omega = 100\ rpm = \frac{2\pi \times 100}{60} = \frac{10}{3}\pi\ rad/s$

$\omega_0 = \frac{2}{3}\pi\ rad/s$

$\omega = \omega_0 + \alpha T$

$\frac{10}{3}\pi = \frac{2}{3}\pi + 0.068 \times T$

$\therefore T = 123.2\ sec.$

(7) A grinding wheel is attached to the shaft of an electric motor of rated speed 1800 rpm. When the power is switched on, the unit attains the rated speed in 5 sec. Assuming uniformly accelerated, determine the no. of revolutions the wheel turns to attain the rated speed.

Solution

$\omega_0 = 0$

$\omega = 1800\ rpm = \frac{1800 \times 2\pi}{60} = \underline{\underline{60\pi\ rad/s}}$

$t = 5\ s$

$\alpha = \frac{\omega - \omega_0}{t} = \frac{60\pi}{5} = 12\pi\ rad/s^2$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 12\pi \times 5^2$

$= \underline{\underline{150\pi\ rad}}$

No. of revolutions $= \frac{150\pi}{2\pi} = \underline{\underline{75\ rev}}$

(8) A wheel rotating at 90 rpm has an angular retardation of 70 rad/min². Find the angular velocity at the end of 30 sec, and the total angular displacement before coming to rest.

Solution

$\omega_0 = 90\ rpm = 90 \times \frac{2\pi}{60} = 3\pi\ rad/s$

$\alpha = -70\ rad/min^2 = \frac{-70}{60 \times 60}$

$\omega = \omega_0 + \alpha t = 3\pi - \frac{70}{60 \times 60} \times 30$

$= \underline{\underline{8.84\ rad/s}}$

Angular displacement before coming to rest;

$\omega_0 = 3\pi;\ \omega = 0.$

$\omega^2 = \omega_0^2 + 2\alpha\theta \Rightarrow 0 = (3\pi)^2 + 2 \times \frac{-70}{60 \times 60} \times \theta \Rightarrow \theta = \underline{\underline{2284.11\ radians}}$

(9) A motarized mortar mixer rotates for 5 sec with a uniform angular acceleration and describes 120 radians during this time. It then rotate with a constant angular velocity and covers 100 rad during the next 5 sec. Calculate the initial angular velocity & angular acceleration

Solution

1st interval

$t = 5\,sec$

$\theta = 120\,rad$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$120 = \omega_0 \times 5 + \frac{1}{2}\alpha \times 5^2$

$48 = 2\omega_0 + 5\alpha$ —(1)

$\omega = \omega_0 + \alpha t$

$20 = \omega_0 + 5\alpha$ —(2)

Solving eq (1) & (2) we get

$\omega_0 = 28\,rad/s$

$\alpha = -1.6\,rad/s^2$

2nd interval

$\theta = 100\,rad$

$\theta = \omega t$ ($\because$ constant angular velocity)

$100 = \omega \times 5$

$\omega = 20\,rad/sec$

(10) A shaft is uniformily accelerated from 10 rps to 18 rps in 4 sec. The ~~shaft~~ shaft continues to accelerate at this rate for the next 8 sec. There after the shaft rotates with uniform angular speed. Find the total time to complete 400 revolutions.

Solution.

$\omega_0 = 10\,rps = 2\pi \times 10 = 20\pi\,rad/s$

$\omega = 18\,rps = 2\pi \times 18 = 36\pi\,rad/s$

$t = 4s$

$\omega = \omega_0 + \alpha t$

$36\pi = 20\pi + \alpha \times 4 \Rightarrow \alpha = 4\pi\,rad/s^2$

(At $t = 4$ sec; $\theta = \omega_0 t_1 + \frac{1}{2}\alpha t_1^2$

$= 20\pi \times 4 + \frac{1}{2} \times 4\pi \times (4)^2$

$= 112\pi$

No. of rev $= \frac{112\pi}{2\pi} = 56$)

At $t = 12$ sec; $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= 20\pi \times 12 + \frac{1}{2} \times 4\pi \times 12^2$

$= 528\pi$

No of revolutions $= \frac{528\pi}{2\pi} = 264$

Remaining no. of revolutions $= 400 - 264 = 136$

At, $t = 12$ sec; $\omega = \omega_0 + \alpha t$

$= 20\pi + 4\pi \times 12$

$= 68\pi$ rad/s

$\theta = \omega t' = 68\pi t'$

$t' = \frac{136 \times 2\pi}{68\pi} = 4$ sec.

$\therefore$ Total time for 400 rev $= 12 + 4 = 16$ sec

(11) A wheel rotates with a constant retardation due to braking. From $t = 0$ to $t = 10$ sec, it made 300 revolutions. At time $t = 7.5$ sec, its angular velocity was 40π rad/s. Determine: a) the value of constant retardation, b) total time taken to come to rest, c) the total revolutions made till it come to rest.

Solution

$t = 10$ s ; $\theta = 300 \times 2\pi$

$t = 7.5$ s ; $\omega = 40\pi$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$300 \times 2\pi = 10\omega_0 + \frac{1}{2}\alpha(10)^2$

$600\pi = 10\omega_0 + 50\alpha$

$60\pi = \omega_0 + 5\alpha$ —— (1)

At $t = 7.5$ sec; $\omega = \omega_0 + \alpha t$

$40\pi = \omega_0 + 7.5\alpha$ —— (2)

Solving (1) & (2) $\Rightarrow$ $\omega_0 = 100\pi$ rad/s

$\alpha = -8\pi$ rad/s^2

Let T be the total time taken to come to rest

$\omega = \omega_0 + \alpha T$

$0 = 100\pi - 8\pi \times T$

$\therefore T = \underline{\underline{12.5 \text{ sec}}}$

Total revolutions :- $\theta = \omega_0 T + \frac{1}{2}\alpha T^2$

$= 100\pi \times 12.5 - \frac{1}{2} \times 8\pi \times (12.5)^2$

$= \underline{625\pi \text{ rad.}}$

No. of revolutions $= \frac{625\pi}{2\pi} = \underline{\underline{312.5}}$.

(12) A wheel accelerates uniformly from rest to a speed of 180 rpm in 0.5 seconds. It then rotates at that speed for 2 seconds, before decelerating uniformly to rest in 0.3 seconds. How many revolutions does it make during the entire time interval?

Solution.

Case I

$\omega_0 = 0$

$\omega = 180 \text{ rpm} = 180 \times \frac{2\pi}{60} = 6\pi$ rad/s.

$t = 0.5$ sec.

$\omega = \omega_0 + \alpha t$

$6\pi = 0 + \alpha (0.5)$

$\therefore \alpha = 12\pi \ rad/s^2$

$\therefore \theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= 0 + \frac{1}{2} \times (12\pi) \times (0.5)^2$

$= 4.71 \ rad$

Case II.

~~It then rotates~~

The wheel rotates at the same speed for 2 sec (ie, at uniform angular velocity)

$\therefore \omega = 6\pi \ rad/s.$

$t = 2 \ sec.$

$\therefore \theta = \omega t = 6\pi \times 2 = 37.69 \ rad.$

Case III.

The wheel decelerates uniformly & comes to rest in 0.3 sec.

$\therefore \omega = 0$

$\omega_0 = 6\pi \ rad/s$

$t = 0.3 \ s.$

$\omega = \omega_0 + \alpha t$

$0 = 6\pi + \alpha \times (0.3)$

$\therefore \alpha = -20\pi \ rad/s^2$

$\therefore \theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= 6\pi \times 0.3 + \frac{1}{2} \times (-20\pi) \times 0.3^2$

$= 2.827 \ rad$

$\therefore$ Total angular displacement, $\theta = 4.71 + 37.69 + 2.827$

$= 45.227 \ rad.$

Total No. of revolutions $= \frac{45.227}{2\pi} = 7.199$ revolutions.

(13) A swing bridge turn through 90° in 120 sec. The bridge is uniformly accelerated from rest for the first 40 sec, subsequently it turns with the uniform angular velocity for the next 60 sec. Now the motion of the bridge is uniformly retarded for the last 20 sec. Find:
(a) angular acceleration ; (b) maximum angular velocity
(c) angular retardation of the bridge.

Solution

Total angle turned = $90° = \pi/2$

ie, $\pi/2 = \theta_1 + \theta_2 + \theta_3$ —— (I)

where θ_1, θ_2 & θ_3 are the angles turned in 3 intervals.

I st interval

$t_1 = 40\,s$

$\omega_0 = 0$ ($\alpha_1 \Rightarrow$ angular acceleration)

$\therefore \omega = \omega_0 + \alpha_1 t$

$\omega = 0 + 40\alpha_1$

$\therefore \omega = 40\alpha_1$

$\theta_1 = \omega_0 t + \frac{1}{2}\alpha_1 t_1^2$

$= 0 + \frac{1}{2} \times \alpha_1 \times (40)^2$

$\therefore \theta_1 = 800\alpha_1$ —— (1)

2nd interval

In this case, the bridge turns with uniform angular velocity

$\omega = 40\alpha_1$; $t_2 = 60\,s$

$\theta_2 = \omega t_2 = 40\alpha_1 \times 60$

$\therefore \theta_2 = 2400\alpha_1$ —— (2)

~~3rd interval~~

~~The motion is unifor~~

3rd Interval

In this case, the motion is uniformly retarded.

Let $\alpha_2 \Rightarrow$ uniform angular retardation

$t_3 = 20$ sec.

$\omega = 0$

$\omega_0 = 40\alpha_1$

$\omega = \omega_0 + \alpha_2 t_3.$

$0 = 40\alpha_1 + 20\alpha_2$

$\therefore \alpha_2 = -2\alpha_1$

$\theta_3 = \omega_0 t + \frac{1}{2}\alpha_2 t_3^2$

$= 40\alpha_1 \times 20 + \frac{1}{2}(-2\alpha_1) \times (20)^2$

$= 800\alpha_1 - 400\alpha_1 = 400\alpha_1$

$\therefore \theta_3 = 400\alpha_1$ —— (3)

Substituting (1), (2) & (3) in eq (I)

$\pi/2 = \theta_1 + \theta_2 + \theta_3.$

$\pi/2 = 800\alpha_1 + 2400\alpha_1 + 400\alpha_1$

ie, $3600\alpha_1 = \pi/2$

$\alpha_1 = 4.363 \times 10^{-4}$ rad/s²

$\alpha_2 = -2\alpha_1 = -8.73 \times 10^{-4}$ rad/s².

$\therefore$ angular acceleration, $\alpha_1 = 4.363 \times 10^{-4}$ rad/s²

angular retardation, $\alpha_2 = -8.73 \times 10^{-4}$ rad/s².

Maximum angular velocity: $\omega = 40\alpha_1$

$= 40 \times (4.363 \times 10^{-4})$

$= 1.745 \times 10^{-2}$ rad/s²